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# EXERGONOMICS IN EDUCATION

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ABSTRACT

Exergonomics is a mirror image of ordinary Economics, using only exergy expenditures instead of monetary ones. Some examples of optimization by a simple relation of invested exergy and current exergy expenditures, including heat transfer through a wall, an electrical conductor and a thermal insulating wall are recommended for educational purposes.

**Nomenclature**

| | | |
|---|---|---|
| a | = | correlation parameter |
| C | = | correlation parameter |
| D | = | sum of exergy destructions |
| d | = | wall thickness |
| F | = | surface, square |
| J | = | electrical current |
| j | = | current density, investment cost |
| K | = | net exergy coefficient |
| k | = | overall heat transfer coefficient |
| L | = | length |
| m | = | correlation parameter |
| Q | = | heat flow |
| q | = | heat current density |
| T | = | absolute temperature |
| t | = | temperature drop |
| Z | = | main exergonomic criterion |
| $\alpha$ | = | $T_1/T_0$, temperature ratio |
| $\delta$ | = | exergy current density |
| $\varepsilon$ | = | exergy (or its flow with a dot) |
| $\tau$ | = | normative time |
| $\xi$ | = | specific invested exergy |
| $\rho$ | = | mass density |
| $\eta$ | = | efficiency |

Subscripts:
1,2 = sides of a wall
0 = reference state
opt= optimal
d = delivered
con= consumed

## INTRODUCTION

The synthesis of Thermodynamics and Economics does have a great history. The early part of the history has been described by Gaggioli and El-Sayed[1] and Tsatsaronis[2], who proposed the title "Exergoeconomics" at the beginning of eighties. The subsequent papers up to now are published mostly in the Proceedings of International conferences with an acronym ECOS, they are very much respected world-wide, the last is [3]. The detailed description of exergoeconomic methodology involving the costing of exergy expenditures in monetary units has been given by Tsatsaronis in the textbook[4]. This methodology is widely used by many practitioners in a real design.

However, the same practitioners feel a sore point here, the prices uncertainty. As a well known example the oil prices might be mentioned. Within the time span of the design and construction of an ordinary big power plant, in 1973 – 1983, oil prices quadrupled and then dropped to their original value. Imagine a designer in 1973.Which prices ,then, should he use in design ? In the same time, exergy expenditures were much more firm and stable. For instance, the data [16] for construction materials and fuels are stable and valid for a long time. That's why, aside from the well known ExergoECOnomics the much less known branch of exergy analysis, referred to as Exergonomics [10,11] was developed. It is a mirror image of ordinary economics but with the value of every good or service measured in exergy expenditures instead of money. This expenditures are divided on two major parts : invested and current ones.

The approach goes back to Frederick Soddy, who tried to replace money by means of energy expenditures at the beginning of this century[5]. The crucial role of exergy, rather than of energy, is now evident. The use of restrictions of Second Law in cogeneration calculations and costing has been clearly advocated by G.Arons in 1926[6]:

> "…the evaluation of rejected heat not by amount of calories, but by power which it may give passing through an ideal steam engine…"

Then he recommended multiplying the heat flow by the Carnot factor, exactly what we now do in the exergy accounting of cogeneration power plants.

The formulation of target function for exergy accounting in power plant optimization (including the invested and current parts) was presented in Ref. [7,p.367] :

> "…before any lower bound …has been reached, one bounces the minimum of the total exergy requirement …exergy of capital and fuel/heat".

Authors[7] mentioned the Soddy's attempt and its systematic elaboration by Chenery[8]. The presentation of total specific exergy expenditure as the sum of inversed exergy efficiency and net-exergy coefficient with its minimization in general form might be found in [9, 14] .It is also reproduced in the present paper.

While exergy expenditure alone may by no means be the basis for decision-making, neither may money. Real life forces us to take into account pollution, time restrictions and other factors . That's why Exergonomics might be considered as an auxiliary tool, to prove the solutions, made on other basis. For more reliable decision making the simultaneous optimization by at least three target functions (exergy-money-pollution) is needed. It is a more difficult task, not discussed here.

At present it is an accepted fact that no one decision in Engineering is possible without a computer simulation of the problem. The more the computer is involved, however, the more the logic of the solution remains vague. The computer never takes the place of logic. This is especially true in education, where some logic should be transferred to the students. For educational purposes simplified problems should be constructed which admit a simple straightforward solution.

The paper is aimed at presenting a clear model of a link between the invested and current exergy expenditures, which let us find an optimal exergy efficiency, and some simple examples of exergonomic optimization, recommended for educational purposes .

### The Main Criterion of Exergonomics

When considering exergy efficiency we should clearly define what is our system, where is its boundary at the entrance and at the exit and then divide the outlet (delivered) exergy flow by the inlet (consumed) one. If our system actually is a subsystem, a part of a greater system, our partial optimization might not coincide with the optimum of greater system. By comparison of fuel-fired plants the input exergy flow should be fuel. It is not obligatory, if only a part of the unit is optimized.

The total life story of every energy conversion unit is presented on the Fig. 1, where

$\varepsilon_B$ =invested exergy, needed to manufacture the unit.
The total exergy efficiency is the ratio of delivered exergy to the sum of exergy expenditures:

$$\eta_{tot} = 1/( 1/\eta + 1/K) \qquad (1)$$
(1)

where

$\eta$ = ordinary exergy efficiency, based on current exergy flows,

$K = \varepsilon d\ \tau / \varepsilon_B$ = net exergy coefficient, ratio of delivered exergy to invested exergy.
The inverse quantity

$$Z = \frac{1}{\eta_{tot}} = \frac{1}{\eta} + \frac{1}{K} \qquad (2)$$
(2)

is the main criterion in Exergonomics in the same sense as COE (Cost of Energy) is the main criterion in Economics, which is usually subjected to minimization.

This approach is similar to the so called ELCA (Exergy Life Cycle Assessment), see for example [12], however the word "Exergonomics "appeared in [11] a little earlier and the subject differs from ELCA by exergy discounting requirements and some other respects, not mentioned in present paper.

**Invested Exergy Models**

In general, for the arbitrary function $K(\eta)$, the optimal solution is [10,11]:

$$Z_{min} = \left( 1 + \left( -\frac{dK}{d\eta} \right)^{1/2} \right) / K; \qquad \eta_{opt} = K / \left( -\frac{dK}{d\eta} \right)^{1/2} \qquad (3)$$
(3)

which is found by solving $\partial Z/\partial \eta = 0$. The problem consists of finding the particular function $K(\eta)$, reflecting every case study.

The first attempt to find correlation between investment cost and exergy efficiency belongs to Szargut [13]. He selected for the monetary investment cost j the function

$$j = j_0 \frac{\eta}{1 - \eta} \qquad (4)$$
(4)

which gives the right trend, $j \to \infty$ as $\eta \to 1$.

The current approach to cost evaluation of capital investment through exergy expenditures is described in Ref.[4], which also lists many additional references.

An approximation with the two correlation parameters a and m has been proposed in [14]

$$K = a\eta^{-m} \qquad (5)$$

which gives

$$Z_{min} = \left( \frac{m}{a} \right)^{\frac{1}{m+1}} + \frac{1}{a} \left( \frac{a}{m} \right)^{\frac{m}{m+1}}; \qquad \eta_{opt} = \left( \frac{a}{m} \right)^{\frac{1}{m+1}} \qquad (6)$$
(6)

The deficiency of the model of eq.(5) is its inability to reflect the main physical condition of the approach of exergy efficiency to 1, the infinite increase of the size of equipment and, hence, its invested exergy.

Here the model of [15] is recommended as rather simple and relevant: the invested exergy is proportional to the delivered exergy and inversely proportional to the exergy destruction rate:

$$\varepsilon_B = \frac{\dot{\varepsilon}_d \cdot \tau}{C^2 (1-\eta)} \qquad (7)$$

Here $C^2$ is a single correlation parameter needed to adjust (7) to any particular problem. The needed trend $\varepsilon_B \to \infty$ as $\eta \to 1$ is evident.

From Eq. (7) we obtain:

$$K \equiv \frac{\varepsilon_d \tau}{\varepsilon_B} = C^2 (1-\eta) \qquad (8)$$

$$Z = \frac{1}{\eta} + \frac{1}{K} = \frac{1}{\eta} + \frac{1}{C^2 (1-\eta)} \qquad (9)$$

$$\frac{\partial Z}{\partial \eta} = -\frac{1}{\eta^2} + \frac{1}{C^2 (1-\eta)^2} = 0; \qquad \eta_{opt} = \frac{C}{1+C}; \qquad Z_{min} = \left(\frac{C+1}{C}\right)^2 \qquad (10)$$

$$\eta_{tot}^{max} = \eta_{opt}^2 \qquad (11)$$

If Eq. (7) is valid, the maximal total efficiency is equal to the square of the optimal one. Equation (11) seems to be the shortest formula in exergy analysis. On Fig. 2 one can see the curves $\eta_{tot}$ versus $\eta$ and the geometrical place of maxima, forming the square parabola. The three superimposed curves reflect the increase of C.

## DC Electrical Conductor

The case is of interest because it is the simplest example with explicit invested exergy, a complete correspondence of exergy to power. Entropy does not play any role here. Let us consider a direct current power line with given current J, length L, cross-section area F, made of material with electrical conductivity $\sigma$, density $\rho$ and exergy intensity $\xi$ (specific exergy consumption to produce 1 kg of material). The sum of the fuel exergy losses due to electrical resistance and invested exergy to build the conductor is

$$D = \frac{1}{\eta} J^2 \frac{L\tau}{\sigma F} + \xi\rho L F \qquad (12)$$

From $\partial D/\partial F = 0$ we obtain

$$j_{opt} = \frac{J}{F_{opt}} = \left(\frac{\xi\rho\sigma\eta}{\tau}\right)^{1/2} \qquad (13)$$

For an aluminium conductor, $\xi$ = 330 MJ/kg [14], $\rho$ = 2500 kg/m$^3$, $\sigma$ = 2.5 .10$^7$ Ohm$^{-1}$m$^{-1}$, $\tau$ = 10$^9$ s (33 years), $\eta$ = 0.35, and the optimal current density is . It is much less than in common practice (about 1 A/mm$^2$). Note here the power plant efficiency $\eta$ , which reduces electrical exergy to that of fuel.

### Heat Transfer Through a Wall

Two fluid streams of temperatures $T_2$ and $T_1$ are separated by a wall of thickness d, Fig. 3. The total heat flow through the surface F is

$$Q = Fq = Fk\left(T_2 - T_1\right) = Fkt; \qquad t = T_2 - T_1 \qquad (14) \tag{14}$$

The exergy current density from the left is

$$\delta_2 = q\left(T_2 - T_0\right)/\,T_2 = kt\left(T_2 - T_0\right)/\,T_2 \tag{15}$$

The exergy current to the right is

$$\delta_1 = q\left(T_1 - T_0\right)/\,T_1 = kt\left(T_1 - T_0\right)/\,T_1 \tag{16}$$

As $T_1$ is less than $T_2$, $\delta_1$ is less than $\delta_2$. Their difference is just the exergy destruction. The exergy efficiency of the heat transfer process is

$$\eta = \frac{\delta_1}{\delta_2} = \frac{T_1 - T_0}{T_2 - T_0} \cdot \frac{T_2}{T_1} \tag{17}$$

When $t \to 0$ we have $\eta \to 1$; however $q \to 0$ and $F \to \infty$ for any given Q.

Assuming that the wall as flat, or that its curvature radius is much greater than the wall thickness, the invested exergy is

$$\varepsilon_B = Fd\rho\xi \tag{18}$$

The delivered exergy for a normative time is

$$\varepsilon_d = F\delta_1\tau = Fkt\tau\left(T_1 - T_0\right)/\,T_1 \tag{19}$$

while the consumed exergy is

$$\varepsilon_{con} = F\delta_2\tau = Fkt\tau\left(T_2 - T_0\right)/\,T_2 \tag{20}$$

The net exergy coefficient is

$$K = \frac{\varepsilon_d}{\varepsilon_B} = \frac{k\tau t}{d\rho\xi}\,\frac{T_1 - T_0}{T_1} = \beta t\,\frac{T_1 - T_0}{T_1}; \qquad \beta = \frac{k\tau}{d\rho\xi} \tag{21}$$

Finally, the main exergonomic criterion is

$$Z = \frac{T_1 - T_0}{T_1}\left(1 - \frac{T_0}{T_1 + t} + \frac{1}{\beta t}\right) \tag{22}$$

From $\partial Z/\partial t = 0$ we have $(T_1 + t)^2 = \beta T_0 t^2$ and optimal temperature drop

$$\frac{t_{opt}}{T_1} = \left[\pm (\beta T_0)^{1/2} - 1\right]^{-1} \qquad (23)$$

Denoting $\alpha = T_1/T_0$ ,for maximal total efficiency we have

$$\eta_{tot}^{max} = (\beta T_0)^{1/2} \left[ \left[(\beta T_0)^{1/2} - 1\right] \left( 1 + \frac{1}{(\beta T_0)^{1/2} (\alpha - 1)} \right) + \frac{\alpha}{\alpha - 1} \right]^{-1} \qquad (24)$$

(24)

The behaviour of maximal efficiency and $t_{opt}$ is presented in Fig. 4. It is clear that a high total efficiency is possible only for a sufficiently high value of the governing dimensionless criterion .

For a more concrete analysis let us introduce the fictitious temperature drop

$$t_f = \frac{1}{\beta} = \frac{d\rho\xi}{k\tau} \qquad (25)$$

This is needed to transfer the heat flow $d\rho\xi/\tau$ to supply the energy embodied in the wall for the time $\tau$. Now the criterion

$$(\beta T_0)^{1/2} = (T_0 / t_f)^{1/2} \qquad (26)$$

takes the form of the root of the ratio of reference temperature to fictitious drop.

The numerical data for a shell-and-tube counterflow high pressure air heater with inlet temperature 450 K, d = 1 cm, k = 50 W/m$^2$K, $\xi$ = 100 MJ/kg for steel tube, $\rho = 10^4$ kg/m$^3$ and life time 10years by 5500 hours/year are $(\beta T_0)^{1/2} = 17.3$, $\alpha = 1.5$, $t_{opt}$ = 27.6 K,  = 0.816. If high pressure air is heated up to 600 K, then $\alpha = 2$, $t_{opt}$ = 36.8 K, and  = 0.90.

So far, we have considered the case $T > T_0$. Another case is refrigeration ($T < T_0$), cf. Fig. 3. Heat flows as before, from left to right, however the exergy current is reversed. Repeating the preceding analysis we have

$$\eta = \frac{T_0 - T_2}{T_2} \cdot \frac{T_1}{T_0 - T_1}; \qquad K = \frac{T_0 - T_2}{T_2} \frac{k\tau}{d\rho\xi} t \qquad (27)$$

The main criterion is

$$Z = \frac{T_2}{T_0 - T_2} \left( \frac{T_0}{T_2 - t} - 1 + \frac{1}{\beta t} \right) \qquad (28)$$

From $\partial Z/\partial t = 0$ we have

$$t_{opt} = T_2 \left[ 1 \pm (\beta T_0)^{1/2} \right]^{-1} \qquad (29)$$

The minus sign makes no physical sense, because $t > T_2$ is impossible. For a typical refrigerating recuperator at 180 K, made of aluminum tubes with $\rho$ = 3000 kg/m$^3$, d = 2 mm, $\xi$ = 300 MJ/kg, k = 1000 W/m$^2$K and $\tau$ = 10 years we have $(\beta T_0)^{1/2} = 180$, $t_{opt}$ = 1 K, which is in agreement with common practice. Note, that k is the overall heat transfer coefficient, which includes the thermal resistance of the wall itself ($d/\lambda$) and convective heat transfer on both surfaces, $\alpha_1$ and $\alpha_2$.

**Thermal Insulation Optimization**
An absolute thermal insulation is impossible. Heat leakage exists as soon as a temperature difference exists. Sometimes the engineering task is to diminish this leakage regardless to expenditure, as it is the case in attempts to reach absolute zero. However, in industry and architecture it is important to determine the reasonable level of thermal insulation. For us the word "reasonable" means the least sum of exergy, lost through the thermal insulation, and exergy, spent to manufacture this insulation (see Fig. 5). Here the specific heat current is

$$q = k\,(T_2 - T_0) \qquad (30)$$

The exergy current lost through insulation is

$$\delta = q\,\frac{T_2 - T_0}{T_2} = k\,(T_2 - T_0)^2 / T_2 \qquad (31) \qquad (31)$$

The overall heat transfer coefficient is

$$k = \left(\frac{1}{\alpha_1} + \frac{d}{\lambda} + \frac{1}{\alpha_2}\right)^{-1} \qquad (32)$$

The sum of the lost exergy and the invested exergy is

$$D = \delta\tau + d\rho\xi \qquad (33)$$

$$\frac{\partial D}{\partial d} = -\frac{(T_2 - T_0)^2}{\lambda T_2}\,\frac{\tau}{[(1/\alpha_1) + (d/\lambda) + (1/\alpha_2)]^2} + \rho\xi = 0 \qquad (34)$$

$$d_{opt} = \frac{\lambda(T_2 - T_0)}{(\lambda T_2 \rho\xi/\tau)^{1/2}} - \left(\frac{\lambda}{\alpha_1} + \frac{\lambda}{\alpha_2}\right) \qquad (35)$$

$$q_{opt} = \left(\frac{1}{\tau}\lambda T_2 \rho\xi\right)^{1/2} \qquad (36)$$

The most important application of these results seems to be in civil engineering: how to select the optimal wall thickness. Here are the two numerical examples:

(1) Brick wall for a building: $T_2$ = 295 K, $T_0$ = 273 K, $\lambda$ = 1 W/mK, $\rho$ = 2640 kg/m$^3$, $\xi$ = 5 MJ/kg, $\tau$ = 50 years, $q_{opt}$ = 49.7 W/m$^2$, 0.24 < $d_{opt}$ < 0.44m for $10 \le \alpha_{1,2} \le 1000$W/m$^2$K.

(2) Thermal insulation of glass wool for a dry ice (solid carbon dioxide) storage, $T_2$ = 200 K, $T_0$ = 295 K, $\lambda$ = 0.03 W/mK, $\rho$ = 50 kg/m$^3$, $\xi$ = 30 MJ/kg $\tau$ = 40 years. Here the results $q_{opt}$ = 2.67 W/m$^2$ and $d_{opt}$ = 1.06 m do not depart from the common practice.

**Conclusion**
It is possible to develop a routine procedure to find the trade-off between invested exergy and current exergy expenditure. This procedure can be described analytically in very simple terms, and can be used for educational purposes.

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FIG. 1 The history of an energy unit, including construction and operation time.

FIG. 2 Total efficiency versus exergy efficiency, .

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FIG. 4 Maximal total efficiency versus the governing criterion $(\beta\ T_0)^{1/2}$, and optimal temperature drop versus the same criterion.

FIG. 5 Exergy losses through a thermally insulated wall.

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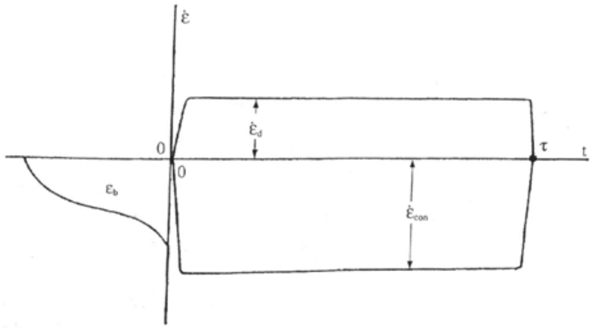


Fig. 1. The history of an energy unit, including construction and operation time.

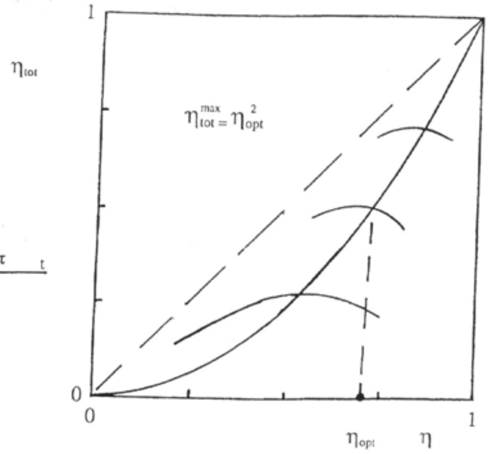


Fig. 2. Total efficiency versus exergy efficiency.

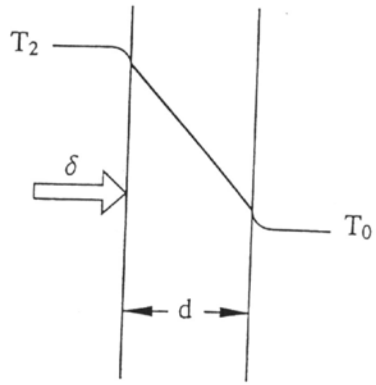


Fig. 5. Exergy losses through a thermally insulated wall.

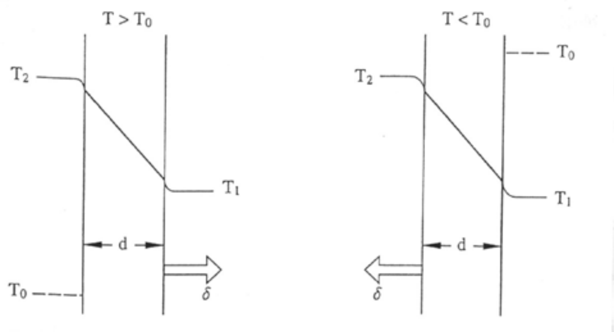


Fig. 3. Heat transfer through a wall for a power plant ($T>T_0$) and refrigerator ($T<T_0$).

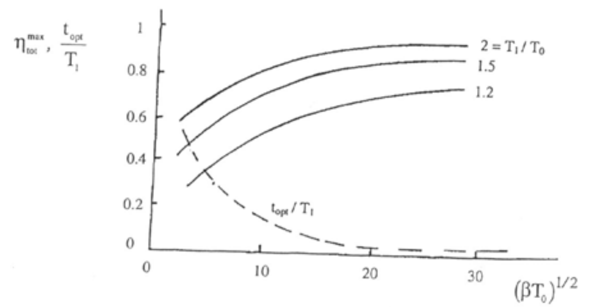


Fig. 4. Maximal total efficiency versus the governing criterion $(\beta\ T_0)^{1/2}$, and optimal temperature drop versus th same criterion.